**2022 May Day MCM**

Problem A. Ordering and biological learning of Vascular Robotics

With the development of micro electro mechanical systems(MEMS), humans have been able to process smaller and smaller machines.A vascular robotic is a machine small enough to be put into a blood vessel to treat a disease.Vascular robotic can carry drugs into blood vessels to treat diseases related to blood vessels at designated points, and can also act as a vascular scavenger to remove viruses and keep people healthy.Therefore, vascular robotic has attracted more attention.

There are many types of vascular robotics, among which ABLVR is used in a hospital.This vascular robotic has two main characteristics:①Can be assembled.The robotic has a container boat (similar to a submarine) that is motivated and swims in the blood.Four manipulators are mounted around the vessel, similar to humans, with a biological brain and a robotic arm, which is controlled by the biological brain.manipulators can be removed and replaced from the vessel.②Need to study.The vascular robotic replication function without direct information, new purchased un-experienced manipulators before work need to be done in advance biological learning (training), similar to the study of the human brain, in a specific environment, several biological brain chip blank manipulators (novices) are required to learn in the simulation vessel under the guidance of trained manipulators (skilled workers) for a week, until a novice reach the level of skilled workers.

The vascular robotic works in the patient's blood vessel for a week, it must be removed after a week.After take out, removing the manipulator requires a week of maintenance before it can work again, and maintenance is always required if no work is scheduled. Newly purchased container boats need to be checked and adjusted for a week before they can be used, container boats do not have to be maintained after use, they can be used continuously, but they also need to be maintained if not used.It is assumed that the purchased container boat and manipulators arrive at the start of each week and arrange immediately for inspection of commissioning and biological learning (training).The relevant cost data are shown in Annex 1.Annex 2 is the number of vascular robotics required by the hospital at weeks 1-104.

The hospital is expected to have 13 container boats and 50 skilled manipulators before it starts its robotic vascular treatment operation in the first week. Please create a mathematical model to answer the following questions.

Question 1: At the beginning of each week, hospitals can buy manipulators and container boats.Each skilled operator can act as an instructor to "guide" the 10 purchased new manipulators for biological learning.If you only consider weeks 1-8, how many container boats and manipulators are need to buy each week to meet treatment and minimize operating costs?

Question 2: Vascular robotics are at risk of working in patients' blood vessels. When they encounter macrophages, they will be completely destroyed if they cannot avoid them.Assuming that 20% of vascular robotics are damaged every week (The number of damages is rounded off), the other conditions follow question 1. Considering the weeks 1-104 taken together, how many container boats and manipulators should be purchased in total to meet treatment requirements and minimize operating costs? And fill the relevant results in Table 1.In addition, compared and analyzed the results of weeks 1-8 with those of week 1-8 of question 1.

Table 1: Related result data for Question 2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Week | Number of container boats purchased | Number of manipulators purchased | Number of manipulators under maintenance | Number of container boats under maintenance | Number of trained manipulators (including "skilled man" and "novice") | Total cost  (Unit: yuan) |
| Week 12 |  |  |  |  |  |  |
| Week 26 |  |  |  |  |  |  |
| Week 52 |  |  |  |  |  |  |
| Week 78 |  |  |  |  |  |  |
| Week 101 |  |  |  |  |  |  |
| Week 102 |  |  |  |  |  |  |
| Week 103 |  |  |  |  |  |  |
| Week 104 |  |  |  |  |  |  |
| Week 1-104  Total |  |  |  |  |  |  |

Question 3: Assuming 10% of vascular robotics damage per week (The number of damages is rounded off) if each skilled operator can "guide" the number of new manipulators to no more than 20, Same as question2, please find out how many container boats and manipulators need to be purchased in total during the period 1-104 weeks to meet the treatment requirements and minimize the operating costs. Fill relevant results and data in Table 2.

Table 2: Related result data for Question 3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Week | Number of container boats purchased | Number of manipulators purchased | Number of manipulators under maintenance | Number of container boats under maintenance | Number of trained manipulators (including "skilled man" and "novice") | Total cost  (Unit: yuan) |
| Week 12 |  |  |  |  |  |  |
| Week 26 |  |  |  |  |  |  |
| Week 52 |  |  |  |  |  |  |
| Week 78 |  |  |  |  |  |  |
| Week 101 |  |  |  |  |  |  |
| Week 102 |  |  |  |  |  |  |
| Week 103 |  |  |  |  |  |  |
| Week 104 |  |  |  |  |  |  |
| Week 1-104  Total |  |  |  |  |  |  |

Question 4: There is a preferential policy for the purchase of manipulators and container boats, the unit price of container boats is 200 yuan per boat when the one-time purchase quantity is not more than 5. When the number of container boats purchased at one time exceeds 5 but not more than 10, the unit price of the part with more than 5 boats is 180 yuan per boat. When more than 10 container boats are purchased at a time, the unit price of more than 10 containers is 160 yuan per boat. Similarly, the unit price for a one-time purchase of less than 20 manipulators is 100 yuan per unit. When the number of manipulators is more than 20 but less than 40 at a time, the unit price of more than 20 manipulators is 90 yuan per unit. When more than 40 manipulators are purchased at a time, the unit price of more than 40 manipulators is 80 yuan per unit.Other conditions follow question3, how will the total number of container boats and manipulators purchased during weeks 1-104 be adjusted? Fill relevant results and data in Table 3.

Table 3: Related result data for Question 4

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Week | Number of container boats purchased | Number of manipulators purchased | Number of manipulators under maintenance | Number of container boats under maintenance | Number of trained manipulators (including "skilled man" and "novice") | Total cost  (Unit: yuan) |
| Week 12 |  |  |  |  |  |  |
| Week 26 |  |  |  |  |  |  |
| Week 52 |  |  |  |  |  |  |
| Week 78 |  |  |  |  |  |  |
| Week 101 |  |  |  |  |  |  |
| Week 102 |  |  |  |  |  |  |
| Week 103 |  |  |  |  |  |  |
| Week 104 |  |  |  |  |  |  |
| Week 1-104  Total |  |  |  |  |  |  |

Question 5: Predict the need for vascular robotics use during weeks 105-112. In order to study the cost of using the vascular robotics for weeks 105-112, following the conditions of Question 4, two solutions can be considered.

Solutions 1:1-104 weeks on the optimal results, on the basis of the hospital in the 105th week that may need to every 300 yuan to buy can directly use the container boat and every 150 yuan to buy skilled operator, and a week later all according to the number of preferential policies to buy suitable in question 4 new container boat and new handlers, satisfy the blood of 105-112 weeks Manage the needs of robotics.

Solutions 2: Consider the needs of vascular robotics at weeks 1-112.

Please compare the difference between the minimum operating costs for weeks 1-112 between the two options.

**Appendix 1 Costs associated with vascular robotics**

|  |  |
| --- | --- |
| Category | **Price** |
| Container boat | 200 yuan per unit |
| manipulators | 100 yuan per unit |
| Manipulators Maintenance | 5 yuan per unit/week |
| Container boat maintenance | 10yuan per unit/week |
| Manipulators (including "skilled worker") training | 10 yuan per unit |

**Appendix 2 Number of vascular robotics used at weeks 1-104 (**units**)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Week 1-8 | 11 | 5 | 4 | 7 | 16 | 6 | 5 | 7 |
| Week 9-16 | 13 | 6 | 5 | 7 | 12 | 5 | 4 | 6 |
| Week 17-24 | 9 | 5 | 5 | 11 | 29 | 21 | 17 | 20 |
| Week 25-32 | 27 | 13 | 9 | 10 | 16 | 6 | 5 | 7 |
| Week 33-40 | 11 | 5 | 5 | 6 | 12 | 7 | 7 | 10 |
| Week 41-48 | 15 | 10 | 9 | 11 | 15 | 10 | 10 | 16 |
| Week 49-56 | 26 | 21 | 23 | 36 | 50 | 45 | 45 | 49 |
| Week 57-64 | 57 | 43 | 40 | 44 | 52 | 43 | 42 | 45 |
| Week 65-72 | 52 | 41 | 39 | 41 | 48 | 35 | 34 | 35 |
| Week 73-80 | 42 | 34 | 36 | 43 | 55 | 48 | 54 | 65 |
| Week 81-88 | 80 | 70 | 74 | 85 | 101 | 89 | 88 | 90 |
| Week 89-96 | 100 | 87 | 88 | 89 | 104 | 89 | 89 | 90 |
| Week 97-104 | 106 | 96 | 94 | 99 | 109 | 99 | 96 | 102 |